

5.3

Slope



Good skiers enjoy skiing on hills with a greater **slope** because they can go faster. Ski runs are rated on a variety of factors, including the slope, or steepness. The steeper the ski run, the more challenging it is.

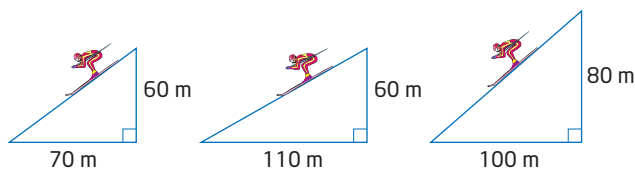
slope

- a measure of the steepness of a line
- calculated as $\frac{\text{rise}}{\text{run}}$

Investigate A

How can you determine the steepness of a hill?

The diagrams represent ski hills.



- Rank the hills in order of their steepness, from least to greatest.
- A hill rises 2 m over a horizontal run of 8 m. A second hill rises 4 m over a horizontal run of 10 m. Which is the steeper hill? Explain.
- Reflect** Describe your technique for determining steepness.



rise

- the vertical distance between two points

run

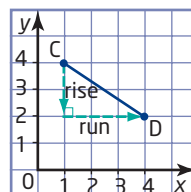
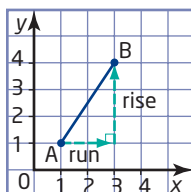
- the horizontal distance between two points

Investigate B

How can you determine the slope of any line segment?

The steepness of a line segment is measured by its slope. The slope is the ratio of the **rise** to the **run** and is often represented by the letter m .

$$m = \frac{\text{rise}}{\text{run}}$$



When you are looking at a graph on a Cartesian grid, read from left to right. A line segment rising from left to right has a positive slope. A line segment falling from left to right has a negative slope.

1. Consider the graph of line segment AB.
 - a) Is the slope positive or negative? Explain how you know.
 - b) Determine the rise and run by counting grid units.
 - c) Determine the slope of the line segment AB using $m = \frac{\text{rise}}{\text{run}}$.
2. Consider the graph of line segment CD.
 - a) Is the slope positive or negative? Explain how you know.
 - b) Determine the rise and run by counting of grid units.
 - c) Determine the slope of the line segment CD using $m = \frac{\text{rise}}{\text{run}}$.
3. a) On a piece of grid paper, set up coordinate axes. Plot the points A(1, 1) and D(5, 1). Join the points to form line segment AD.
 - b) Determine the rise and the run.
 - c) Describe what happens when you calculate the slope of a horizontal line segment.
4. a) On the same set of axes, plot the point E(1, 5). Join points to form line segment AE.
 - b) Determine the rise and the run.
 - c) Do you think it is possible to calculate the slope of a vertical line segment? Justify your answer.
5. **Reflect** Describe how you can find the slope of any line segment.

Example 1 Slope of a Loading Dock

The ramp at a loading dock rises 2.50 m over a run of 4.00 m.

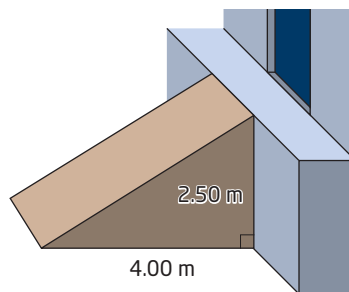
- a) Calculate the slope of the ramp.
- b) Explain the meaning of the slope.

Solution

$$\begin{aligned} \text{a) } m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2.50}{4.00} \\ &= 0.625 \end{aligned}$$

The slope of the ramp is 0.625.

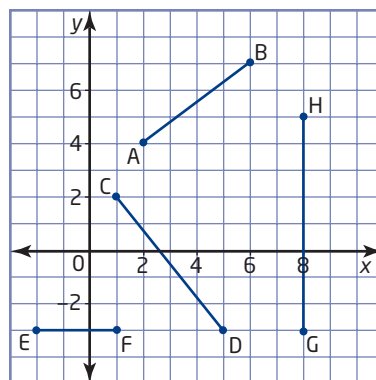
- b) The ramp rises 0.625 m vertically for every 1 m run horizontally.



Example 2 Slope of Line Segments

Calculate the slope of each line segment, where possible. Describe the direction and how it relates to the slope.

- a) AB
- b) CD
- c) EF
- d) GH

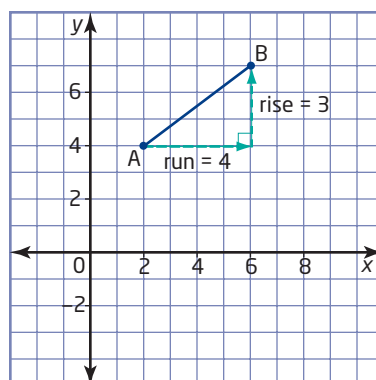


Solution

- a) Count the number of grid units to find the rise and the run.

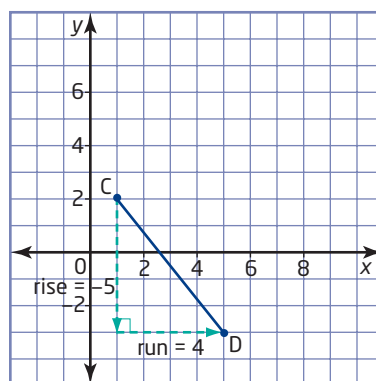
$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{3}{4} \end{aligned}$$

The slope of AB is $\frac{3}{4}$. The direction is up 3 units as you go to the right 4 units.



$$\begin{aligned} \text{b) } m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-5}{4} \\ &= -\frac{5}{4} \end{aligned}$$

The slope of CD is $-\frac{5}{4}$. The direction is down 5 units as you go to the right 4 units. This is why it is negative.



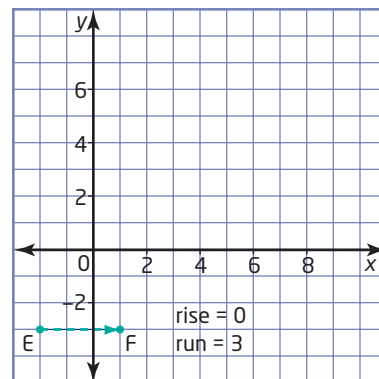
The rise tells me if the direction is up or down. The run tells me if the direction is right or left. So, a rise of 3 means go up 3, and a run of 4 means go right 4.

A rise of -5 means go down 5, and a run of 4 means go right 4.

- c) EF is a horizontal line segment. This line segment has no rise.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{0}{3} \\ &= 0 \end{aligned}$$

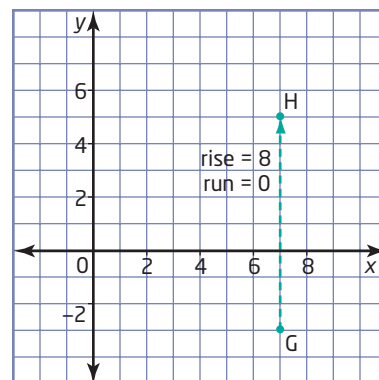
The slope of EF is 0. EF does not have an up or down direction. It is flat. The rise is 0 for a run of 3.



- d) GH is a vertical line segment. This line segment has no run.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{8}{0} \end{aligned}$$

Since division by zero is undefined, the slope of GH is undefined. GH does not have a left or right direction. The run is 0 for a rise of 8.



Example 3 Use the Slope to Find a Point

A line segment has one endpoint, A(4, 7), and slope of $-\frac{5}{3}$. Find the coordinates of another possible endpoint, B.

Solution

Method 1: Draw a Graph

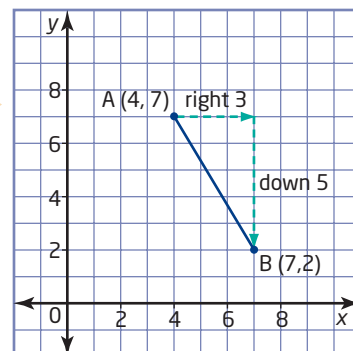
Plot the point A(4, 7).

Use the slope $-\frac{5}{3}$ to find another point.

Another possible endpoint is B(7, 2).

$$\text{Since } -\frac{5}{3} = \frac{-5}{3} = \frac{\text{rise}}{\text{run}},$$

I will go to the right 3 and down 5.



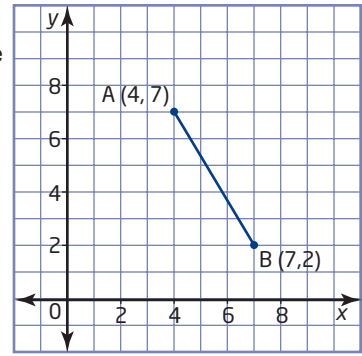
There is an infinite number of solutions. What if I had used a rise of 5 and a run of -3? or 10 and -6? or -10 and 6?

Method 2: Use the Coordinates

The run is 3 and the rise is -5 . Add these values to the x - and y -coordinates, respectively, of point A.

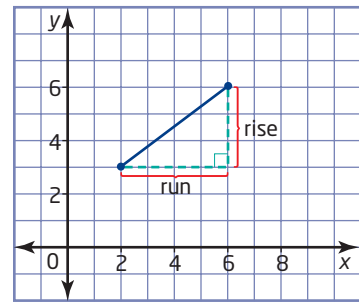
$$(4 + 3, 7 + (-5)) = (7, 2)$$

Another possible endpoint is B(7, 2).



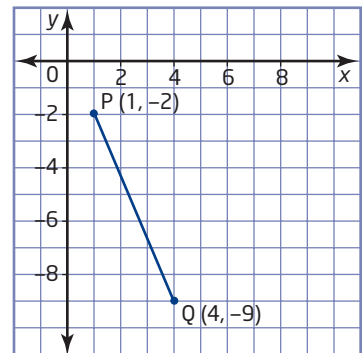
Key Concepts

- The slope, m , is a measure of the steepness of a line segment. It is calculated as $m = \frac{\text{rise}}{\text{run}}$.
- A line segment rising from left to right has a positive slope.
- A line segment falling from left to right has a negative slope.

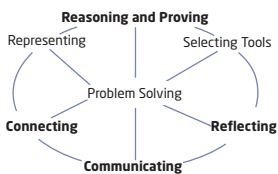
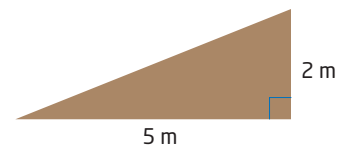


Communicate Your Understanding

- C1** Kelly looked at this line segment and concluded that the slope had to be negative because the coordinates of the points contained negative numbers. Is her reasoning correct? Explain.



- C2** A ramp rises 2 m over a run of 5 m.
- How would you change the rise to make the slope steeper?
 - How would you change the run to make the slope steeper?

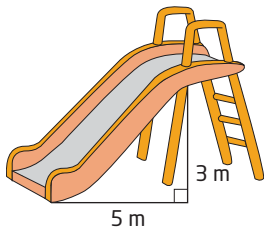


Practise

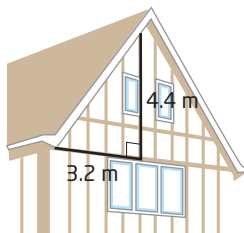
For help with questions 1 to 3, see Example 1.

1. Determine the slope of each object.

a)



b)



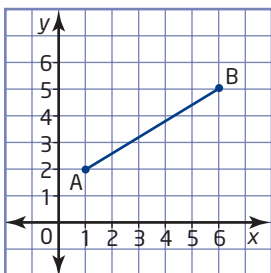
2. A section of road is built with a vertical rise of 2.5 m over a horizontal run of 152 m. Find the slope, to the nearest hundredth.
3. To be safe, a wheelchair ramp needs to have a slope no greater than 0.08. Does a wheelchair ramp with a vertical rise of 1.4 m along a horizontal run of 8 m satisfy the safety regulation?

For help with questions 4 and 5, see Example 2.

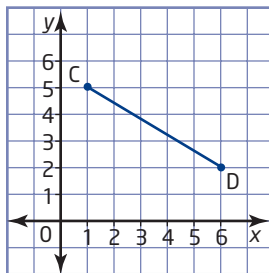
4. For each line segment,

- count grid units to find the rise
- count grid units to find the run
- determine the slope

a)

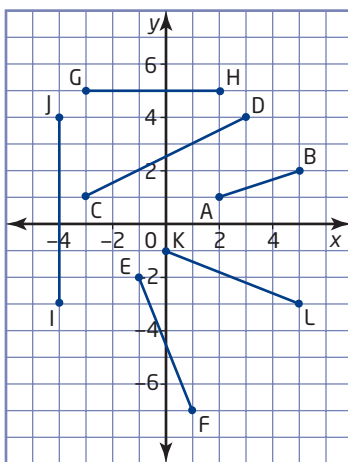


b)



5. Calculate the slope of each line segment, where possible.

- a) AB
b) CD
c) EF
d) GH
e) IJ
f) KL



For help with questions 6 and 7, see Example 3.

6. A line segment has one endpoint of A(3, 1).

a) Plot the point A on a grid.

b) Use the slope $\frac{3}{2}$ to locate another possible endpoint B.

What are the coordinates of point B?

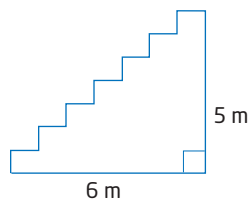
7. A line segment has one endpoint of A(6, -2) and slope of $-\frac{3}{4}$.

Find the coordinates of another possible endpoint B by adding the appropriate values to the coordinates of point A.

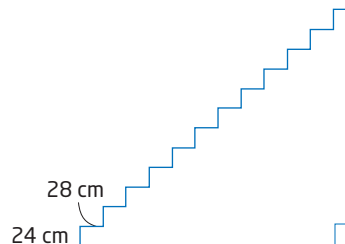
Connect and Apply

8. For safety reasons, a staircase should have a slope of between 0.58 and 0.70. Determine whether each staircase is within the safe range.

a)



b)



9. Given a point A(-2, 5), find the coordinates of a point B so that the line segment AB has each slope.

a) $\frac{2}{3}$

b) $-\frac{2}{3}$

c) 4

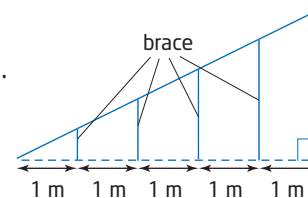
d) -3

e) 0

f) undefined

10. A ramp needs to have a slope of $\frac{3}{5}$.

Determine the length of each vertical brace.



Did You Know?

Saint John, New Brunswick has the steepest main street in Canada. King Street has an 8% grade.

11. Slopes of roads are called grades and are expressed as percents.

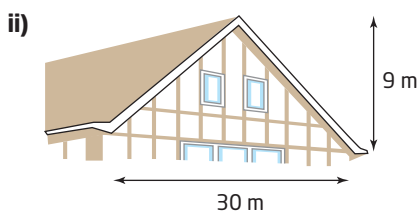
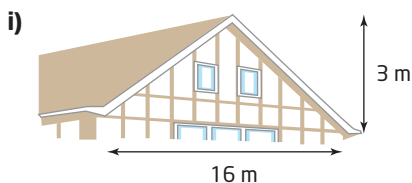
a) Calculate the grade of a road that rises 21 m over a run of 500 m.

b) For a road to have a grade of 3%, how far does it have to rise over a run of 600 m?

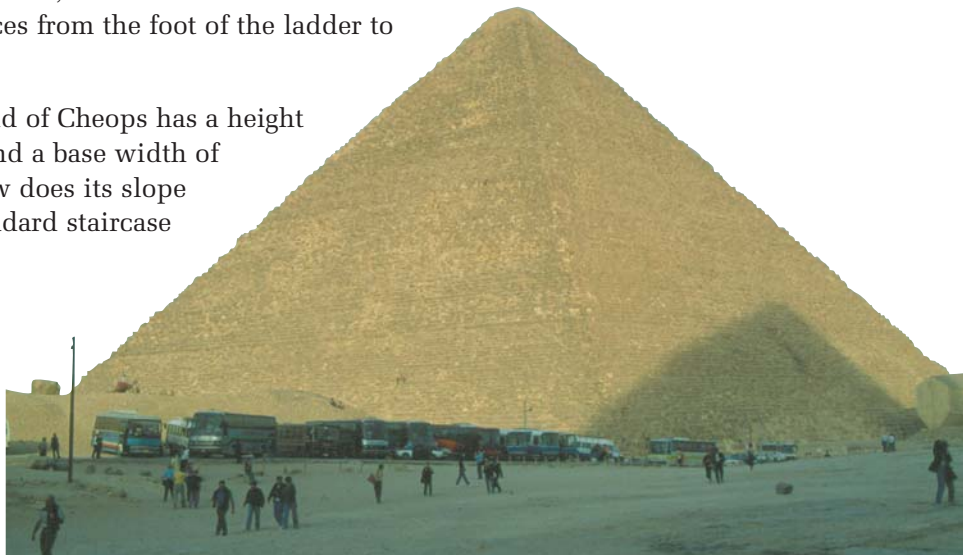
- 12.** Roofers call the slope of a roof its pitch. Roofs have different pitch classifications, which indicate how safe they are for roofers to walk on. They are classified as shown in this table.

Classification	Pitch
Shallow	$m \leq \frac{3}{12}$
Medium	$\frac{3}{12} < m \leq \frac{6}{12}$
Steep	$m > \frac{6}{12}$

- a)** Classify each roof by its pitch.



- b)** A roof is 10 m wide and has a pitch of $\frac{5}{12}$. Find the height.
- 13.** Two ramps are being built with the same slope. The first ramp is twice the height of the second ramp. Does the first ramp have to be twice as long as the second ramp? Explain.
- 14.** A steel beam goes between the tops of two buildings that are 7 m apart. One building is 41 m tall. The other is 52 m tall. What is the slope of the beam?
- 15.** For safety reasons, an extension ladder should have a slope of between 6.3 and 9.5 when it is placed against a wall. If a ladder reaches 8 m up a wall, what are the maximum and minimum distances from the foot of the ladder to the wall?
- 16.** The Great Pyramid of Cheops has a height of about 147 m and a base width of about 230 m. How does its slope compare to a standard staircase with slope 0.7?

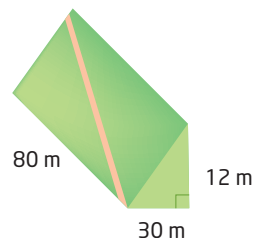


17. In 1967, Montreal hosted Expo 67, an international fair, to celebrate Canada's 100th birthday. Canada's pavilion was an upside-down pyramid called Katimavik, which means *meeting place* in Inuktitut, the language of the Inuit. The base width is about 55 m and the height is about 18 m. Calculate the slope of the sides. Compare the slope of the sides to the slope of the Great Pyramid of Cheops, which you found in question 16.

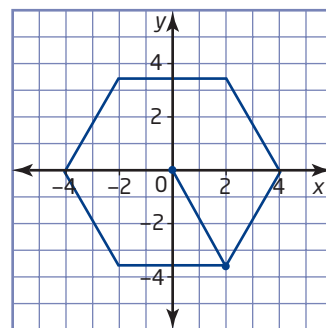


Extend

18. A cross-country ski area classifies its courses based on the range of slopes. If the slopes are less than 0.09, the course is classified as easy. For slopes between 0.09 and 0.18, the course is intermediate. For slopes greater than 0.18, the course is difficult. For a ski hill 10 m tall, what range of horizontal runs is appropriate for each classification?
19. A hiking trail has been cut diagonally along the side of a hill, as shown. What is the slope of the trail?

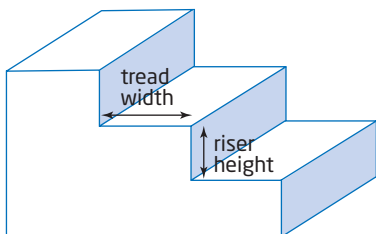


20. A regular hexagon has six sides of equal length. One is drawn on a grid as shown. Determine the slope of the line segment from the centre to the vertex indicated. Explain your reasoning.



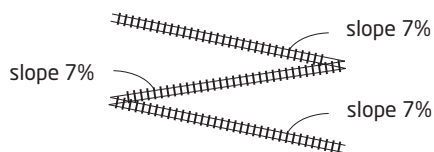
21. How safe are the stairs around your home? To answer this question, carry out the following investigation.

- a) For different sets of stairs around your home, measure the tread width and riser height. Try to get measurements for several sets of stairs. Record your measurements in a table, and compute the slope for each set of stairs. Draw conclusions about which set of stairs is the least safe in your home.



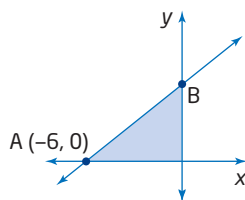
- b) Collect data from at least five classmates. Construct a scatter plot of the data, and draw a line of best fit. Analyse your results and write a report on your findings.

22. Math Contest Railroad trains cannot go up tracks with a grade (slope) greater than 7%. To go over hills steeper than this, the railroad company builds *switchbacks*. How many switchbacks are needed to get to the top of a hill that is 250 m high? Assume that the maximum length of the run is 1 km. Explain your solution.



23. Math Contest The area of the shaded region is 12 square units. What is the slope of the line through AB?

- A $\frac{3}{2}$
 B $-\frac{3}{2}$
 C 4
 D $\frac{2}{3}$
 E $-\frac{2}{3}$



Did You Know?

The steepest railroad in the world is in the Blue Mountains of New South Wales, Australia.

The maximum gradient is 122%.